# Development of Minimum Cost Capacity Model for Power System Component Expansion (MCCM) 

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#### Abstract

This model forms the basis for determining the number of lines of a given specification to be added to an existing power network, in order to correct the line overload problem. The approach first determines the line type to be recommended. The (MCCM) model is used to identify the lines to be augmented as a result of particular line overload. The susceptance across the right of way is determined at minimum cost. Power system stability is a complex concept that has challenged power system engineers for many years. That is stability and control problem are functions of transmission disturbances in terms of overload problems. Therefore for economic reason, power systems are designed to be operated close to their steady state stability limit thereby eliminating overload problems.


Index Terms: Minimum cost capacity model, lines addition, (impedance, reactive, susceptance), network analysis, power system component.

## 1. INTRODUCTION

The choice of an optimal plan for the expansion of a transmission network is a difficult task and no satisfactory solution has yet been found. The size and locations of the generating plans are usually known. However, difference in demand patterns or in generation availability from those expected can change the flows in a network. Nevertheless, the network is expected to transmit power from the generating units to load centers as required. The procedure using a decoupled load flow approach to find the minimum cost capacity additions are required to accommodate, Known changes in demands and generation. An expansion schedule involving linear and dynamic programming methods was produced. The required input data items are the yearly load and generator nodes and magnitude, the initial transmission grid, the cost of losses and the cost of permissible line additions are giving in per mile. The program of Kattenbael et al tests for reliability by solving the yearly load flows for overloads in single outages. [1]

Garver and Adams and Dell have solved a similar problem using linear programming. Puntel et al and Sullivan defined the problem such that the susceptance is installed in the rights of way to minimize the costs of installing transmission line capacity and of penalizing overloads in each of the transmission lines. Minimization is carried out by first computing a gradient vector. The computation uses an adjunct network based on Tellegen's theorem. These concepts have been extended by this work by comparing qualitative and quantitative methods of system
planning. Phase shifters were used where possible to correct line overloads. [2]

In this work, simple methods are used to determine the number of lines of each specification to be added to a network to eliminate system overloads at minimum cost. The coherency approach developed by Bennon et al is used as a yardstick to determine which lines should be augmented as a result of overloads in some lines. Then a static optimization procedure based on the steepestdescent algorithm determines the new admittance to be implemented along these rights of way. The method is expected to reduce the size of alternative planning networks with considerable savings in cost. An overall economic analysis is not carried out except for adding lines minimum cost to eliminate overloads. [3]

## Symbol

$\varphi_{i}{ }^{\text {Max }} \quad$ Maximum phase angle difference in line
$\mathrm{Y}_{\mathrm{k}} \quad$ Admittance of line K
$\mathrm{X} \quad$ Impedance of a line
N Number of overloaded lines in the system
M Number of rights of way to the augmented
$\mathrm{C}_{1} \quad$ Unit line costs
L Number of lines required for addition in a particular right of way.
$\phi \quad \Psi-\Psi^{\max }$

## 2. MATHEMATICAL FORMULATION:

The mathematical formulation of the problem and assumption can be stated as follows:

- From a given list of possible line addition the line which results in the smallest phase angle difference should be
recommended for addition to the network. This approach has been suggested in previous study for rights of way with more than one line.
- Determine for particular line overload(s), which rights of way should be augmented to alleviate or eliminate the line overload(s) problems.
- Using the steepest descent procedure we can determine the new admittance across these rights of way such lines are added at minimum cost overloads eliminated.
- Determine the number of lines needed for minimum cost elimination of overload which result in the smallest phase angle difference. [4]


## 3. CHOICE OF LINE TYPE

For each line $i$, determine given by the product of line rating and reactance with the smallest $\Psi_{i}$ is chosen for additions to the network. [5]
Determine the rights way to be augmented with the coherency relationship used by Bennon et al the rights of way to be augmented are determined by assuming a linear relationship between the transmission capacity and the admittance. This relationship is given by

$$
\begin{aligned}
\Psi_{k} & =e^{T} y^{-1} e_{i} y_{i} \Psi_{i} \\
q & \left.=\frac{\partial \psi_{i}}{\partial y_{i}} \right\rvert\, \frac{\partial \psi_{k}}{\partial y_{k}}
\end{aligned}
$$

Where $\frac{\partial \psi_{i}}{\partial y_{i}}=e^{T} y^{-1} e_{i} \Psi_{i}$

$$
\begin{equation*}
\frac{\partial \psi_{k}}{\partial y_{k}}=e^{T} y^{-1} e_{k} \Psi_{k} \tag{2}
\end{equation*}
$$

Dividing equation (1) $\mathrm{x}(2)$
We have

$$
\begin{equation*}
=\frac{\frac{\partial \Psi_{k}}{\partial y i}}{\frac{\partial \Psi_{k}}{\partial y_{k}}}=\frac{e^{T} y^{-1} e_{i} \Psi_{i}}{e^{T} y^{-1} e_{i} \Psi_{k}}=q \tag{3}
\end{equation*}
$$

This means that:

$$
q=\frac{\partial \Psi_{k}}{\partial y_{i}} / \frac{\partial \Psi_{k}}{\partial y_{k}}=\frac{e^{T} y^{-1} e_{i} \Psi_{k}}{e^{T}-y^{-1} e_{i} \Psi_{k}}
$$

From equation (1) which determines the effect of capacity changes in line $i$ on the power flow in line $k$.
For a line $i$ terminating at busbars $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ we have


For $i=\mathrm{k}$, q from equation (1) is equal to 1.0. The rights of way to be augmented preferentially are those with $\mathrm{q} \geq 1.0$ which has been that the component with higher values of $q$ will have considerable effect on line and for capacity changes in line $i$. [6]

These phase differences are computed using the DC approximation of the load flow with initial generation and consumption value but different line parameters depending on the line addition. [7]

Static optimization technique
Once the rights of way to be augmented have been determined as in the previous case, the static optimization problem can be stated as follows. [8]

Minimize the phase angle difference $\phi_{\mathrm{k}}$ for the overloaded line K so that the overloads are cleared by changing by changing the admittance of line $i$. Make these changes by adding lines at minimum cost. Thus we want to minimize: [9]

$$
\begin{equation*}
J=\sum_{K=1}^{N}\left(\psi_{k}-\psi_{k}^{\max }\right)+\sum_{i=1}^{M} C_{i} y_{i} \tag{6}
\end{equation*}
$$

For $N$ overloads and $M$ rights of way to be augmented subject to:
The satisfaction of the DC load flow equations
$y \geq 0$

The absolute value of $\Psi$, i.e. $|\Psi|$, is the relevant quantity here. Equation (3) can be rewritten as:
$J=J_{1}+J_{2}$
The gradient vector is also given as:

$$
\begin{equation*}
\frac{\partial J}{d y}=\frac{d J_{1}}{d y}+\frac{d J_{2}}{d y} \tag{7}
\end{equation*}
$$

$\partial J_{2} / \mathrm{dy}_{i}$ is a constant $\left(\mathrm{C}_{i}\right)$ which is supplied with the line specification, while
$\frac{d J_{1}}{d y}=\frac{d \phi}{d y}$
Where $\phi=\Psi-\Psi^{\max }$. It should be noted that $\Psi$ and $\Psi^{\max }$ described in previous case, are not necessarily equal. In matrix form:

$$
\frac{d J}{d y}=\left[\begin{array}{lc}
\partial \phi(1, & 1) / \partial y_{1}+C_{1}  \tag{8}\\
\partial \phi(2, & 2) / \partial y_{2}+C_{2} \\
\vdots & \vdots \\
\partial \phi(M, & M) / \partial y_{M}+C_{M}
\end{array}\right]
$$

Note that

$$
\begin{equation*}
\frac{\partial J}{\partial y_{1}}=\sum_{K=1}^{N} \frac{\partial \psi_{k}}{\partial y_{1}}+C_{i} \tag{9}
\end{equation*}
$$

Thus we get the new vector

$$
\begin{equation*}
y_{1}=y-\alpha \frac{\partial J}{\partial y_{1}} \tag{10}
\end{equation*}
$$

$\alpha$ is some scalar parameter that controls the magnitude of the step. An engineering judgment is required in the choice of $\alpha$. Its objective is to ensure that the minimization of $J$ along a given direction, too small a value of $\alpha$ causes too much iteration to be carried out to clear the overload, although convergence characteristic is eventually achieved. However, when $\alpha$ is very large, then the oscillations around the minimum value may occur. $N$ may vary in the course of the steepest-descent algorithm, in which case the overloads may be cleared one after the other. [10]


Figure 1: Sample Network

Table 1: System Bus Data

| Bus | Power Injections MW |
| :--- | :--- |
| 1. | 1620 |
| 2. | -140 |
| 3. | 0.0 |
| 4. | 1300 |
| 5. | -1940 |
| 6. | 840 |
| Slack bus (Bus 3) |  |

Table 2: Line Data

| Line | From Bus | To <br> Bus | XP.U | Capacity <br> P.U |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 2 | 0.008 | 5.0 |
| 2 | 1 | 3 | 0.020 | 2.0 |
| 3 | 1 | 5 | 0.020 | 5.0 |
| $4^{*}$ | 1 | 6 | 0.005 | 15.0 |
| 5 | 3 | 4 | 0.020 | 2.0 |
| 6 | 4 | 5 | 0.005 | 27.0 |
| 7 | 5 | 6 | 0.030 | 1.0 |

* Double line represented by a single line Assumed data

Table 3: Specifications of candidate lines for addition

| Type | $\mathbf{X}$ <br> $\mathbf{( P . U )}$ | Capacity <br> $\mathbf{( P . U )}$ | Capacity <br> $\mathbf{X}$ <br> $\mathbf{( P . U )}$ | $\mathbf{C}_{\boldsymbol{i}}$ |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 0.02 | 5.0 | 0.10 | 0.1 |
| 2 | 0.01 | 1.0 | 0.01 | 0.1 |
| 3 | 0.03 | 2.0 | 0.06 | 0.1 |

Base MVA $=100.00$
Table 4: Coherence values of the lines

| Line | Coherence |
| :--- | :--- |
| $1-2$ | 0.0000 |
| $1-3$ | 0.0700 |
| $1-5$ | 1.4121 |
| $1-6$ | -0.4250 |
| $3-4$ | 0.0770 |
| $4-5$ | 0.1174 |
| $5-6$ | 1.0000 |

## 5. CONCLUSION

A simple technique formulation has been presented in this work for strengthen a network to alleviate line overloads at minimum cost. The satisfactory performance of the method depends on the appropriate choice of $\alpha$. However, instead of adding many lines in 1-5 as shown above, lines that have higher voltage may be substituted instead.

Recommendation

## Analysis for illustration for load forcast for generation capacity

In a large system the daily load curve during a time of rapid early morning load increase displayed the following loads in GW during successive five minute intervals.

| Time (Mins) | Load (GW) |
| :--- | :--- |
| 46 | 19.30 |
| 47 | 19.78 |
| 48 | 19.96 |
| 49 | 20.31 |
| 50 | 20.69 |

It is true that a straight line approximation would suffice to describe the duration of periods of the form $y=a+b x$. We can calculate the best values for coefficients a and b by least squared estimation and load forecast during period 55minutes.

## Case 1:

For the line $y=a+b x$ through data points $(x, y)$ define the error for any point ( $\mathrm{x}, \mathrm{y}$ ) as:
$\mathrm{e}_{i}=\mathrm{y}_{\mathrm{i}}-\mathrm{a}-\mathrm{bx}_{i}$
The sum of squares of the error value
For N points is

$$
\begin{align*}
& J=\sum_{i=1}^{N} e i e i \\
& \mathrm{~J}=\sum_{i=1}^{N}(y i-a-b x i)(y i-a-b x i) \tag{15}
\end{align*}
$$

Using regression analysis

$$
\begin{align*}
& \Sigma y_{i}^{2}-2 a \quad \Sigma y_{i}-2 b \Sigma x y_{i}=0  \tag{16}\\
& N a^{2}+b^{2} \quad \Sigma x i^{2}+2 a b \Sigma x_{i}=0 \tag{17}
\end{align*}
$$

Taking partial derivatives of J with respect to a and b .

$$
\begin{align*}
& \frac{\partial J}{\partial a}=-\Sigma y_{i}+2 N a+2 b \Sigma x i  \tag{18}\\
& \frac{\partial J}{\partial b}=-2 \Sigma x_{i} y_{i}+2 a \Sigma x i+2 b \Sigma x i^{2} \tag{19}
\end{align*}
$$

For best values of $a$ and $b$ set

$$
\begin{align*}
& \frac{\partial J}{\partial a}=\frac{\partial J}{\partial b}=0 \\
& \Sigma y_{i}=N a+b \Sigma x i \\
& \Sigma x_{i} y_{i}=a \Sigma x i+b \Sigma x i^{2} \tag{20}
\end{align*}
$$

Which are the normal equations to be solved for $a$ and $b$.
Table 5: Sum of square of data

| $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{X}^{2}$ | $\mathbf{X Y}$ |
| :--- | :--- | :--- | :--- |
| 46 | 19.30 | 2116 | 887.80 |
| 47 | 19.78 | 2209 | 929.66 |
| 48 | 19.96 | 2304 | 958.08 |
| 49 | 20.31 | 2401 | 995.19 |
| 50 | 20.69 | 2500 | 1034.50 |
| 240 | 100.04 | 11530 | 4805.23 |
| $\Sigma y_{i}=N a+b \Sigma x i$ |  |  |  |

$$
\begin{aligned}
& \therefore 100.04=5 a \quad+240 b \\
& 4805.23=240 a+11530 b
\end{aligned}
$$

$$
a=4.12 \quad, \quad b=0.33
$$

Forest for $\mathrm{x}=55$

$$
\begin{aligned}
& Y=4.12+0.33 \times 55 \\
& =22.27 \mathrm{GW}
\end{aligned}
$$

Generating capacity (GW) $=22.27$ GW
Case 3: Appling the simplifying assumptions

- DC Load flow method
$/ \mathrm{V}_{1} /=/ \mathrm{V}_{2} /=1.0$
$\mathrm{Y}_{\text {shunt }}$ is neglected as well as $\mathrm{Q}_{2}(=\mathrm{IV} \sin \phi=0)$

Solving for a and b


Figure 2: Simple two bus system representation
$\mathrm{V}_{1}<\alpha_{1}$

$$
\begin{equation*}
Q_{i}=\left|V_{i}\right| \sum_{j=1}^{n}\left|V_{j}\right| \ddot{Y} \sin \left(\alpha_{i}-\alpha_{j}+\theta y\right) \tag{23}
\end{equation*}
$$

$\mathrm{V}_{2}<\alpha_{2}$
$\mathrm{X}_{21}=j 0.0872$

$$
\begin{equation*}
Y i_{j}=Y i j \quad \exp (-\theta \ddot{y}) \tag{24}
\end{equation*}
$$

$\mathrm{b}_{21}=1 / X_{\cdot 21}=11.47$
$P_{21}=11.47\left(\alpha_{2}-\alpha_{1}\right)=-0.6=11.47 \alpha_{2}=$

$$
\alpha_{2}=\frac{-0.6}{11.47}=-0.523 \mathrm{rad}
$$

In the absence of loses $\mathrm{P}_{1}=0.6 \mathrm{pu}$
Case 4: Newton Raphson method

$$
\left[\begin{array}{c}
\Delta P_{2}  \tag{21}\\
\Delta G j
\end{array}\right]\left[\begin{array}{ll}
\frac{\partial P_{2}}{\partial \alpha_{2}} & \frac{\partial P_{2}}{\partial\left|V_{2}\right|} \\
\frac{\partial Q_{2}}{\partial \alpha_{2}} & \frac{\partial Q_{2}}{\partial\left|V_{2}\right|}
\end{array}\right]\left[\begin{array}{l}
\Delta \alpha_{2} \\
\Delta\left|V_{2}\right|
\end{array}\right]
$$

From the load flow equations
$Y_{12}=\frac{1}{j 0.0872}=-j 11.47$
$Y_{12}=j 11.47=11.47\left\llcorner-\Pi / 2 \Rightarrow \theta_{12}=\Pi / 2\right.$
$Y_{22}=-j 11.47+j 0.022 b \div-j 11.45 \Rightarrow \theta_{22}=\Pi / 2$
$\frac{\partial P_{2}}{\partial \alpha_{2}}=-\left|V_{1}\right|\left|V_{2}\right| Y_{12} \sin \left(\alpha_{2}-\alpha_{1}+\theta_{12}\right)$

## Iteration 1:

$$
\left.\frac{\partial P_{2}}{\partial \alpha_{2}}\right|_{0}=-11.47 \sin -\Pi / 2=11.47
$$

$$
\begin{equation*}
P_{i}=\left|V_{i}\right| \sum_{j=1}^{n}\left|V_{j}\right| \mid \ddot{Y} \cos \left(\alpha_{i}-\alpha_{j}+\theta y\right) \tag{22}
\end{equation*}
$$

$\frac{\partial Q_{2}}{\partial \alpha_{2}}=\left|V_{1} \| V_{2}\right| Y_{12} \cos \left(\alpha_{2}-\alpha_{1}+\theta_{12}\right)=0.0$
$\frac{\partial P_{2}}{\partial\left|V_{2}\right|}=2\left|V_{2}\right| Y_{22} \cos \theta_{22}+\left|V_{1}\right| Y_{12} \cos \left(\alpha_{2}-\alpha_{1}+\theta_{12}\right)=0.0$
$\frac{\partial Q_{2}}{\partial\left|V_{2}\right|}=2\left|V_{2}\right| Y_{22} \sin \theta_{22}+\left|V_{1}\right| Y_{12} \sin \left(\alpha_{2}-\alpha_{1}+\theta_{12}\right)=$

### 11.43

The Jacobian matrix:

$$
\begin{aligned}
& {[J]=\left[\begin{array}{ll}
\frac{\partial P_{2}}{\partial \alpha_{2}} & \frac{\partial P_{2}}{\partial\left|V_{2}\right|} \\
\frac{\partial Q_{2}}{\partial \alpha_{2}} & \frac{\partial Q_{2}}{\partial\left|V_{2}\right|}
\end{array}\right]} \\
& {[J]^{[ }=\left[\begin{array}{ll}
11.47 & 0.0 \\
0.0 & 11.43
\end{array}\right]} \\
& P_{2}^{(0)}=\left|V_{1}\right|\left|V_{2}\right| Y_{12} \cos \left(\alpha_{2}-\alpha_{1}+\theta_{12}\right)+\left|V_{2}\right|^{2} Y_{22} \cos \left(\alpha_{2}-\alpha_{1}+\theta_{12}\right) \\
& \quad=0.0
\end{aligned}
$$

Similarly $\mathrm{Q}_{2}{ }^{(0)}=-0.02$
$\Delta \mathrm{P}_{2}{ }^{(0)}=\mathrm{P}_{2}-\mathrm{P}_{2}{ }^{(0)}=-0.6$
$\Delta \mathrm{Q}_{2}{ }^{(0)}=\mathrm{Q}_{2}-\mathrm{Q}_{2}{ }^{(0)}=-0.3+0.02=-2.28$
$\left[\begin{array}{ll}J_{1} & J_{2} \\ J_{3} & J_{4}\end{array}\right]\left[\begin{array}{l}\Delta \alpha \\ \Delta V\end{array}\right]=\left[\begin{array}{l}\Delta P \\ \Delta Q\end{array}\right]$
$\left[\begin{array}{lc}11.47 & 0 \\ 0 & 11.43\end{array}\right]\left[\begin{array}{l}\Delta \alpha^{(0)} \\ \Delta V^{(0)}\end{array}\right]=\left[\begin{array}{l}-0.6 \\ -2.28\end{array}\right]$
$\left[\begin{array}{l}\Delta \alpha^{(0)} \\ \Delta V^{(0)}\end{array}\right]=\left[\begin{array}{l}-0.05244 \\ -0.0244\end{array}\right]$
$\left|V_{2}\right|^{(i)}=\left|V_{2}\right|^{(D)}+\left|\Delta V_{2}\right|^{(D)}=1.0-0.0244=0.9756$
$\left|\Delta \alpha_{1}\right|^{(i)}=\left|\alpha_{2}\right|^{(0)}+\left|\Delta \alpha_{2}\right|^{(D)}=0.0-0.05244=-0.05244$

## Iteration 2:

$\left[J_{1}\right]_{=}\left[\begin{array}{lr}11.18 & -0.601 \\ -0.05 & 10.85\end{array}\right]$
$\mathrm{P}_{2}{ }^{(1)}=0.586=>\Delta \mathrm{P}_{2}{ }^{(1)}=\mathrm{P}_{2}-\mathrm{P}_{2}{ }^{(1)}=-0.014$
$\mathrm{Q}_{2}{ }^{(1)}=-0.295=>\Delta \mathrm{Q}_{2}{ }^{(1)}=\mathrm{Q}_{2}-\mathrm{Q}_{2}{ }^{(1)}=-0.005$

Hence:
$\left[\begin{array}{l}\Delta \alpha \\ \Delta\left|V_{i}\right|\end{array}\right]=\left[J_{1}\right]^{-1}\left[\begin{array}{l}-0.014 \\ -0.005\end{array}\right]=\left[\begin{array}{l}0.00129 \\ -0.00044\end{array}\right]$
$\left|V_{2}\right|^{(2)}=0.9756-0.00044=0.97516$
$\Delta \alpha_{2}^{(2)}=-0.05244+0.00129=-0.05115 \mathrm{rad}$

## Case 5:

bus 2


Figure 3: 3-bus system with all parameters in per unit
Example to show how technique works:
The above 3-bus system with all parameters in per unit on a system with base of 100 MVA . Computing using the load flow technique:
i. The voltage angles
ii. The MW flow in each circuit
iii. The percent loading in each circuit assuming the MW capacity to be 75 MW

$$
\begin{aligned}
& X_{12}=0.08=>b_{12}=\frac{1}{0.08}=12.5 \\
& X_{13}=0.13=>b_{13}=\frac{1}{0.13} \quad=7.69
\end{aligned}
$$

The parallel combination of the circuit on buses 2 and 3.

$$
X_{23}=\frac{(0.06)(0.06)}{0.06+0.06}=0.03 \Rightarrow b_{23}=\frac{1}{0.03}=33.33
$$

The bus susceptance matrix $\left[\mathrm{B}_{\text {bus }}\right\}$

$$
\left[\begin{array}{ccc}
20.19 & -12.5 & -7.69 \\
-12.5 & 45.83 & -33.33 \\
-7.69 & -33.33 & 41.02
\end{array}\right]
$$

This is a singular matrix and as the slack bus angle is fixed at zero degree; all the elements corresponding to bus 1 are eliminated:

$$
\begin{aligned}
& {\left[B_{\text {reduced }}\right]=\left[\begin{array}{lr}
45.83 & -33.33 \\
-33.33 & 41.02
\end{array}\right]} \\
& \text { If } \quad \text { B. } \alpha=\Delta \mathrm{P} \\
& \therefore
\end{aligned} \quad \alpha=\mathrm{B}^{-1} \Delta \mathrm{P} \text { : } \begin{aligned}
& \text { I }
\end{aligned}
$$

$\Delta \mathrm{P}_{2}=-0.45$
$\Delta \mathrm{P}_{3}=1.00-0.75=0.25$

From B. $\alpha=\Delta \mathrm{P}$
$\left[\begin{array}{l}\alpha_{2} \\ \alpha_{3}\end{array}\right]=\left[\begin{array}{lr}45.83 & -33.33 \\ -33.33 & 41.02\end{array}\right]^{-1}\left[\begin{array}{l}-0.45 \\ 0.25\end{array}\right]=\left[\begin{array}{l}-0.013 \\ -0.0046\end{array}\right] \mathrm{rad}$

## Line flows

Line: $\mathrm{P}_{12}=\mathrm{b}_{12}\left(\alpha_{1}-\alpha_{2}\right)=12.5(0.0+0.013)$

$$
=0.1625 \mathrm{pu}
$$

$1-2=16.25 \mathrm{MW}$
Line: $\mathrm{P}_{13}=\mathrm{b}_{13}\left(\alpha_{1}-\alpha_{3}\right)=7.69(0.0+0.0046=$
0.0354 pu
$1-3=3.54 \mathrm{MW}$
Line: $P_{23}=b_{23}\left(\alpha_{2}-\alpha_{3}\right)=33.33(-0.013+0.0046)$
$2-3=-0.2799 \mathrm{pu}=-27.99 \mathrm{MW}$
The negative sign means that the flow is actually from line 3 to 2. The flow in each circuit is 13.995 MW. i.e.

$$
\frac{27.99 \mathrm{MW}}{2}=13.995 \mathrm{MW}
$$

\% loading (with a capacity of 75MW):
Line 1- 2: $\frac{16.25}{75} \times 100 \%=21.66 \%$

Line 1 - 3: $\frac{3.54}{75} \times 100 \%=4.72 \%$
For each line $2-3: \frac{13.995}{75} \times 100 \%=18.66 \%$
As the \% loadings are less than 100 , this means there no are overloaded lines.

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